Computing in Alfonso s Universe

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Alfonso X, “El Sabio,”
king of Castile, León and Galicia 1252–1284

Alfonso s complaint

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(I wasn t consulted either.)

Who ordered *that*?

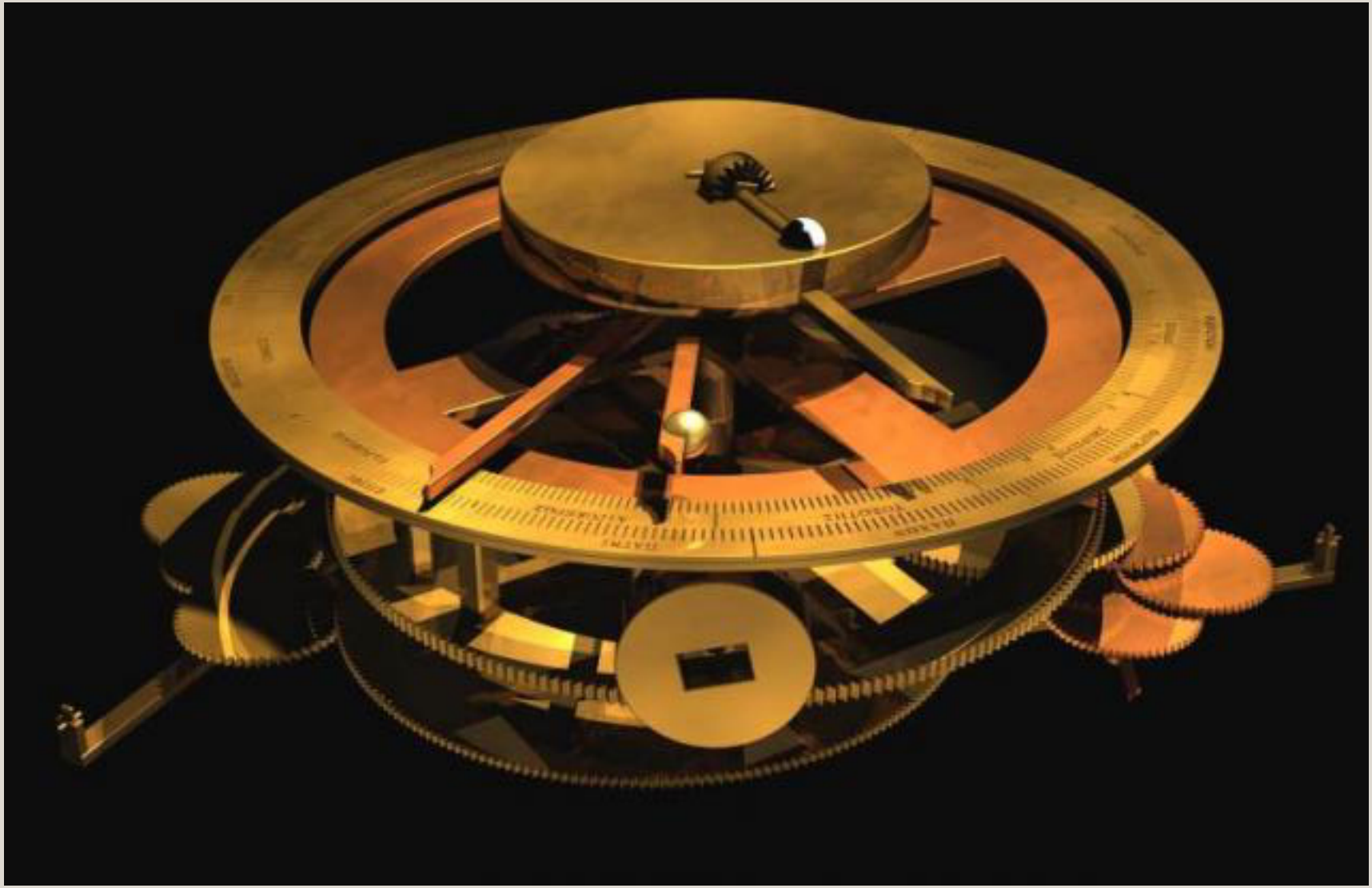
The world seems to be more complicated than it really needs to be. Or, at least, we can imagine a simpler world. Why such gratuitous dif culties?

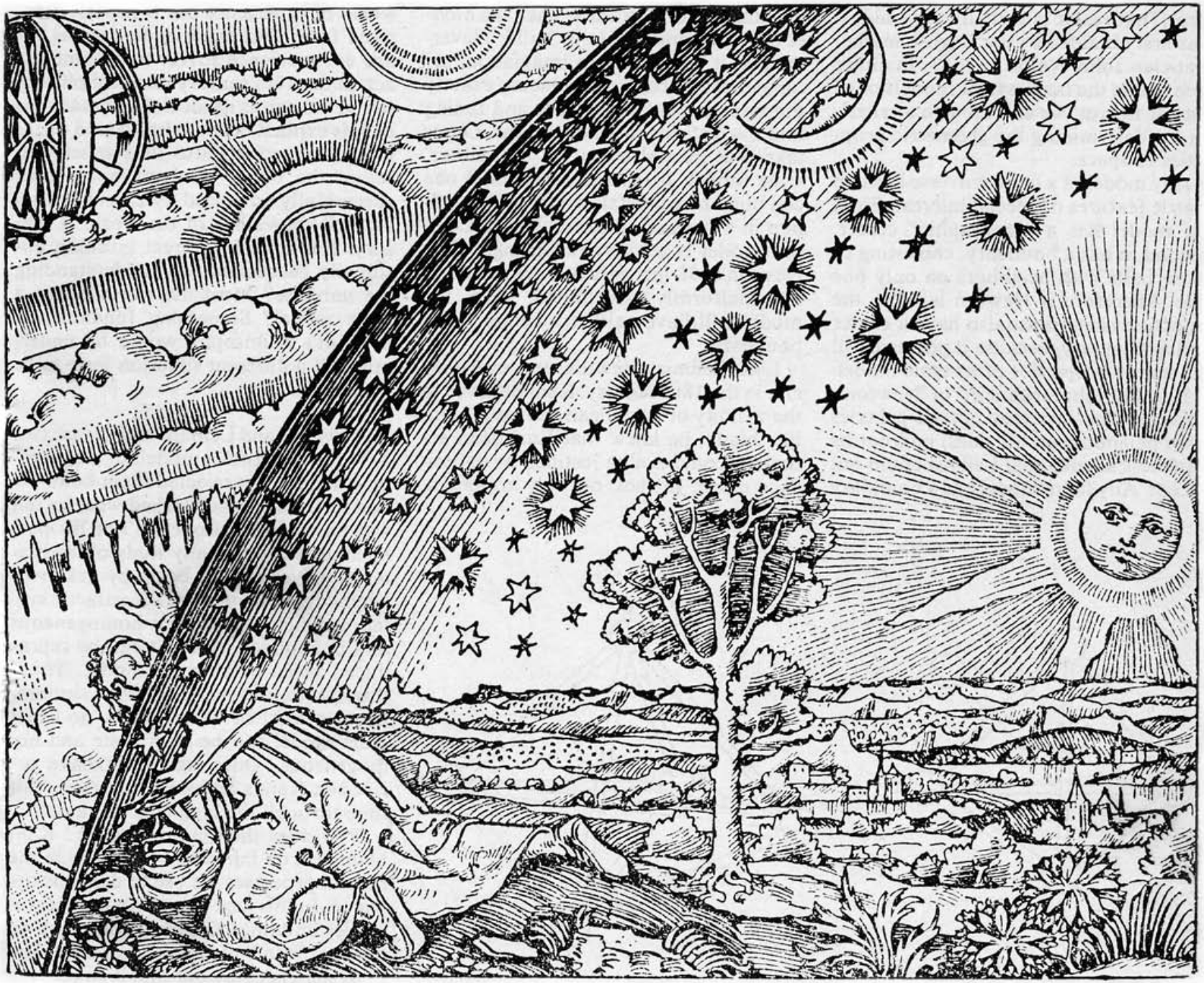
If science is to make any sense, we need a universe with fairly simple laws, which we can hope to understand.

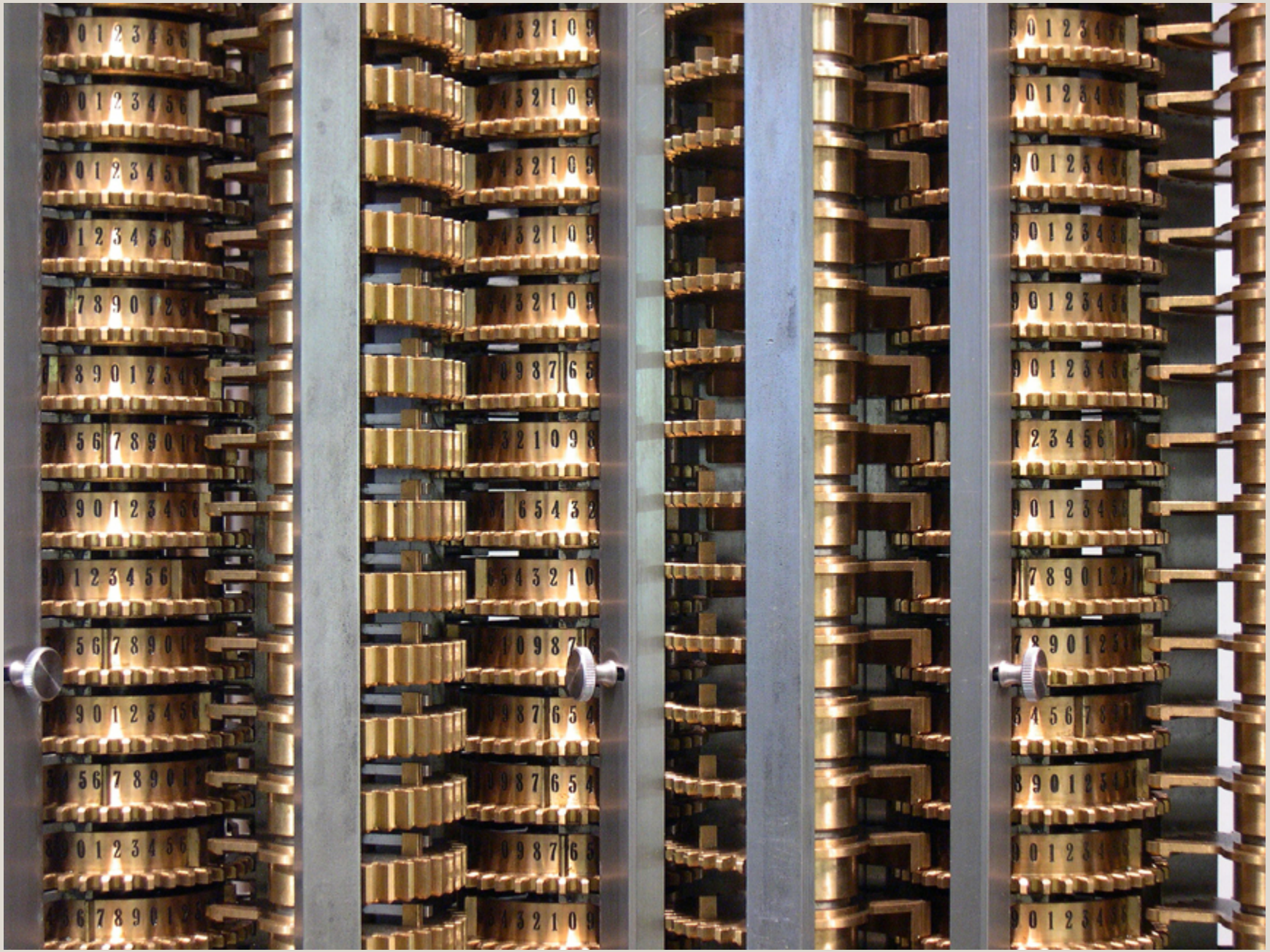
Ultimately, we need laws we can compute with.

(How does the world compute itself?)

The Clockwork Universe







The Computational Universe

“ In a sense, nature has been continually computing the “next state” of the universe for billions of years; all we have to do—and, actually, all we *can* do—is “hitch a ride” on this huge ongoing computation, and try to discover which parts of it happen to go near to where we want. ”

—Tommaso Toffoli

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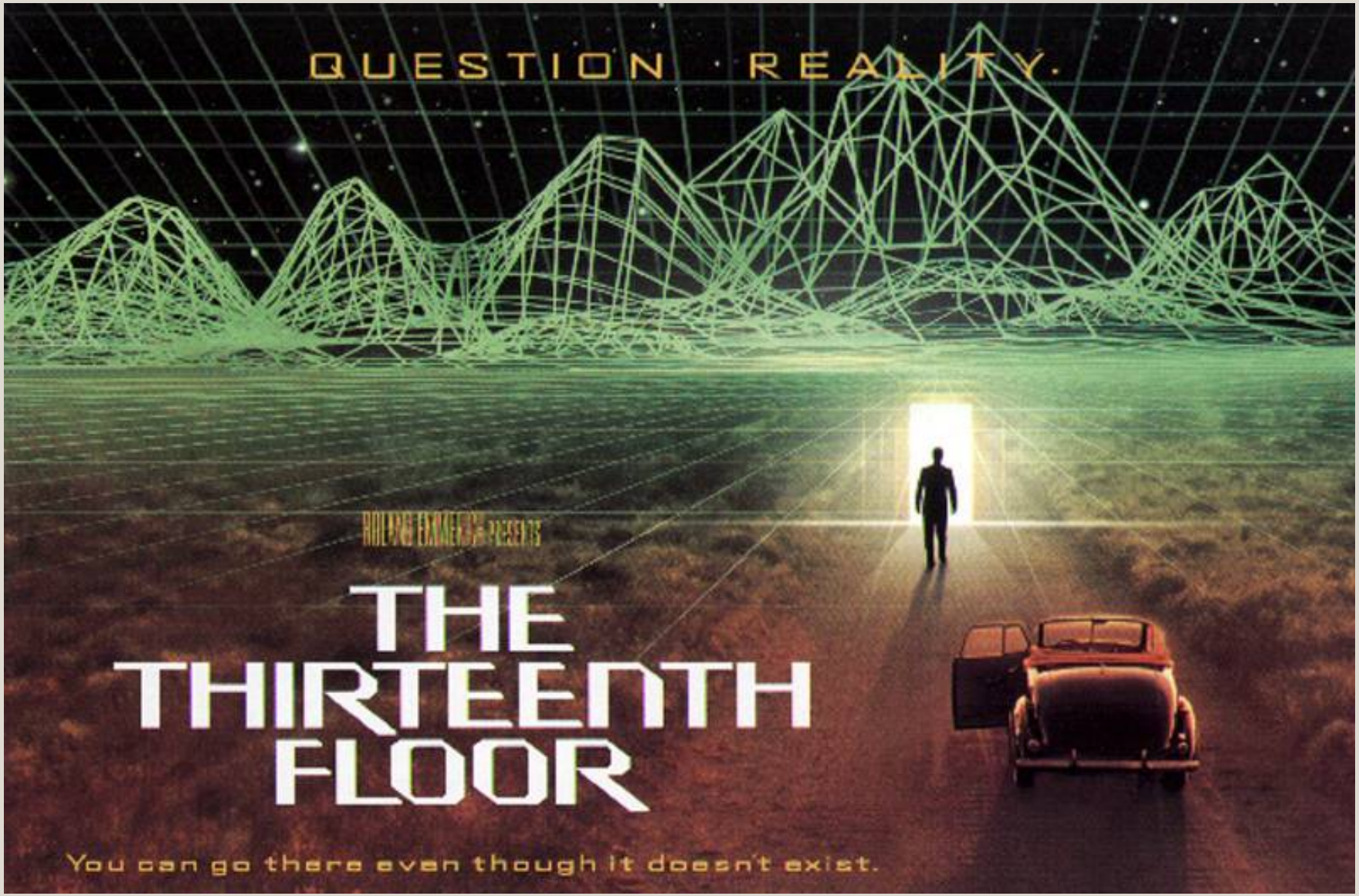
(But sometimes it s hard to catch a ride.)

What this talk is *not* about

- *Not* $P = NP$ and combinatorial problems
- *Not* computational modeling and simulation
- *Not* virtual reality and butter y dreams

(Again: How does the world compute itself?)

It s *just* a metaphor . . .



Ballistic motion

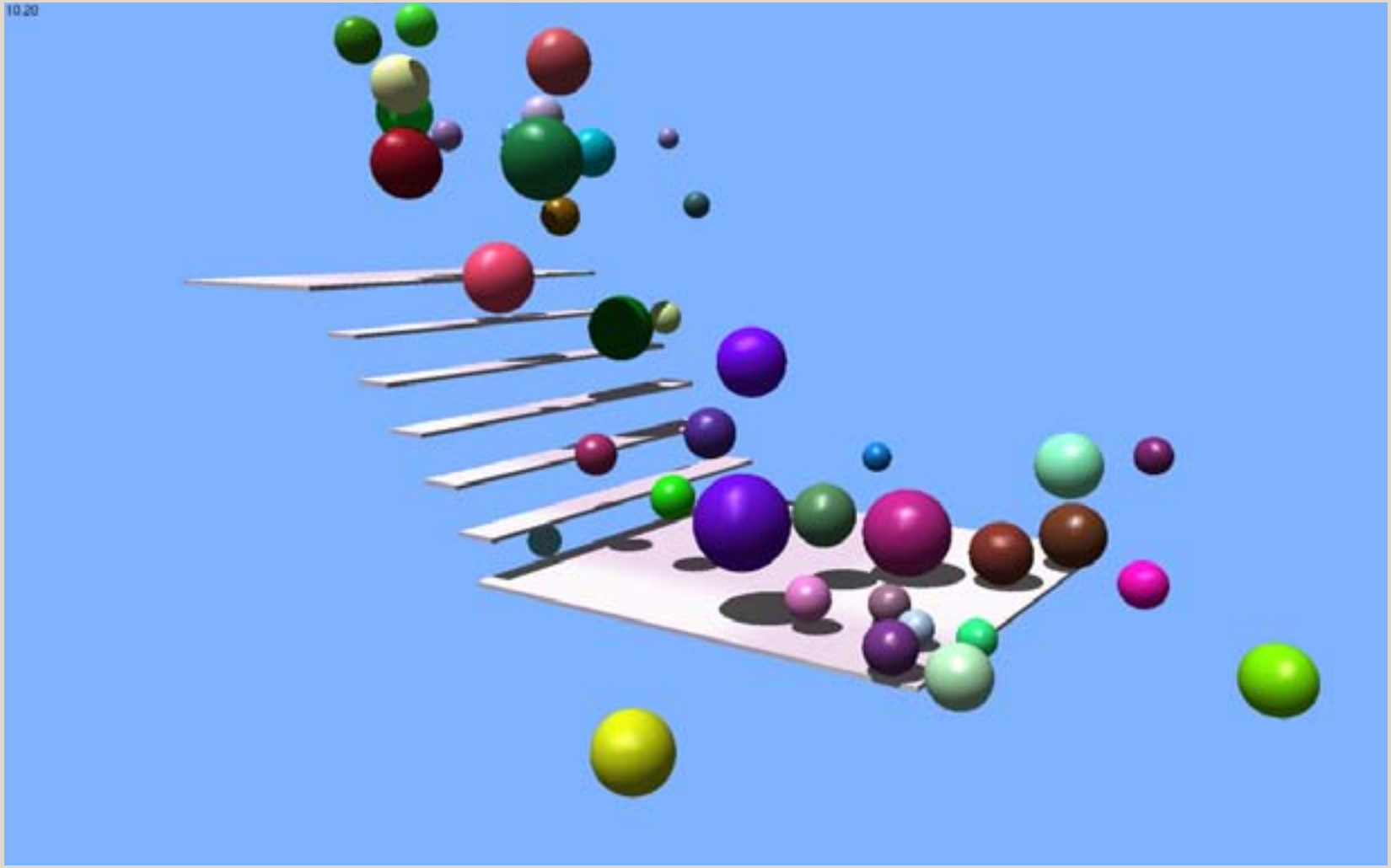
- The Physics 101 version: cannonballs in a vacuum
- Easy: $s = v_0 t + \frac{1}{2} a t^2$
- Even easier:

```
while true do {
    x += v*dt;
    v += a*dt;
}
```

Ballistic motion

- Linear complexity: $\mathcal{O}(n)$
- Per-particle complexity is $\mathcal{O}(1)$
- If every particle computes its own path, the universe runs at constant speed

The collision problem



The *n*-body problem

- Quadratic complexity: $\mathcal{O}(n(n-1)/2)$
- Need as many CPUs as particle *pairings*
- Extreme nonlocality: Every particle talks to every other, no matter how distant
- Hard to intuit how nature does this computation

N-body approximations

- Hierarchical schemes yield $\mathcal{O}(n \log n)$
- (Disputed claims of $\mathcal{O}(n)$)
- Even if $\mathcal{O}(n)$, no obvious mapping to physical structures
- Nature doesn t approximate

Newton on the *n*-body problem

“ The orbit of any one planet depends on the combined motion of all the others, not to mention the actions of all these on each other. To consider simultaneously all these causes of motion and to de ne these motions by exact laws allowing of convenient calculation exceeds, unless I am mistaken, the forces of the entire human intellect. ”

—Isaac Newton

Who cares how fast the universe runs?

Adding one particle to an *n*-particle universe increases the computational burden by a factor of *n*. There s a burst of star formation in M82, and over here in the Milky Way the lights dim and the world slows down.

But so what? We can t measure “the speed of time,” and if we suppose the universe is a computer, we can t detect uctuations in its clock rate.

Really, who *cares*?

“ God does not care about our mathematical dif culties. He integrates empirically. ”

—Albert Einstein

“ Nature is not embarrassed by dif culties of analysis. ”

—Augustin Fresnel

I care

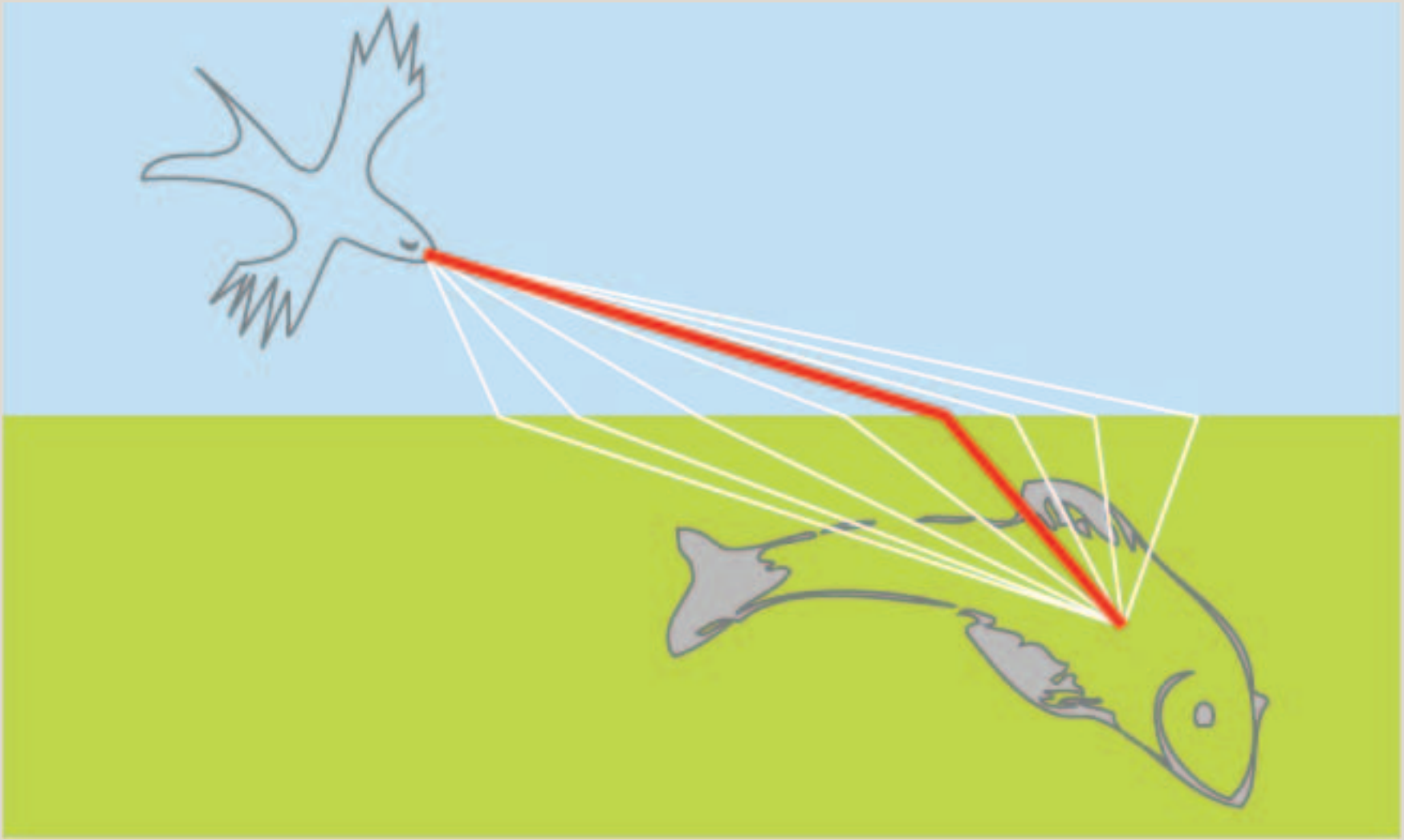
Is it too much to ask that the most elementary aspects of the world be readily computable?

If we can t compute it, can we really say we understand it?

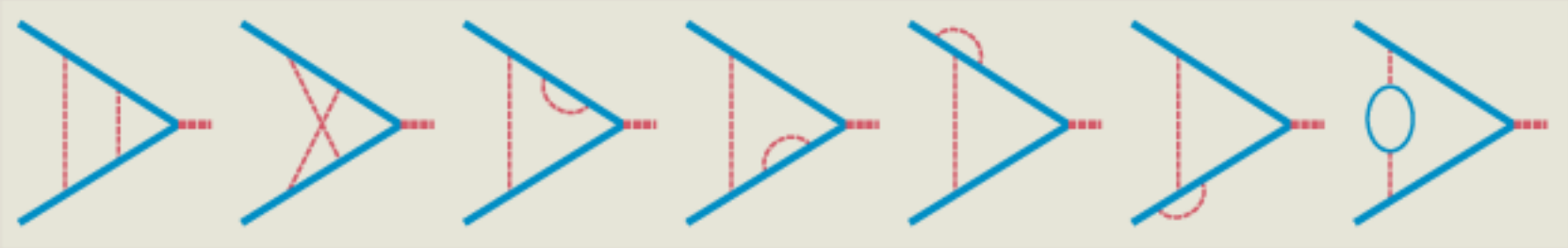
“ I am standing on the threshold about to enter a room. It is a complicated business… I must make sure of landing on a plank travelling at twenty miles a second round the sun—a fraction of a second too early or too late, the plank would be miles away. I must do this whilst hanging from a round planet, head outward in space, and with a wind of aether blowing at no one knows how many miles a second through every instice of my body. ”

—Arthur S. Eddington

Worse news: The principle of least action



Still worse news: Quantum field theory



The quantum world

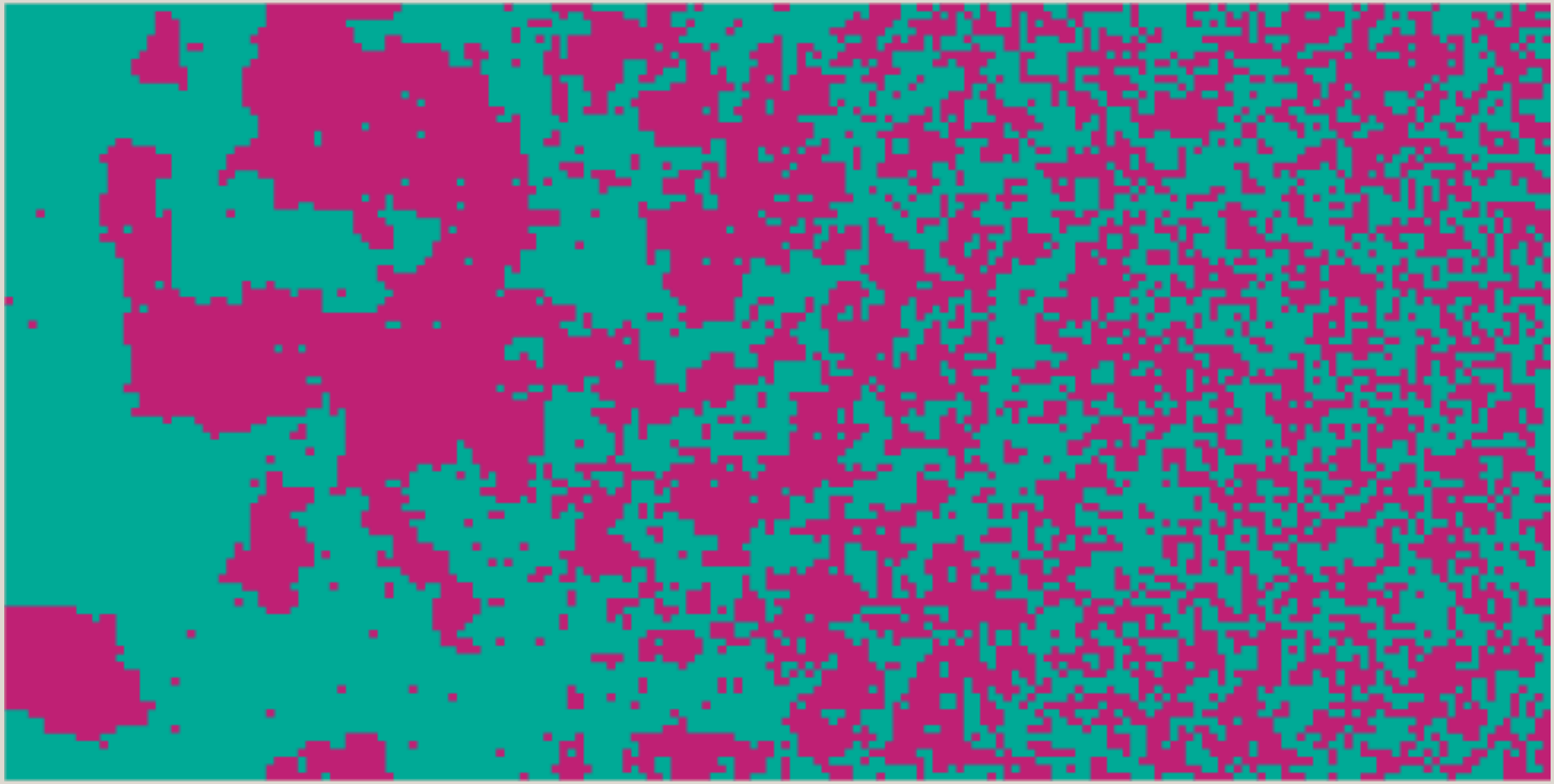
Is a quantum-mechanical universe compatible with a deterministic model of computation?

If the state of a single particle is speci ed by m variables, then the state of n particles requires not mn but m^n variables. Almost anything you try to compute, the algorithm is exponential.

How do we get out of this mess?

- Give up. Admit that the universe is not computable, or at least that it does not allow a simple evolution algorithm
- Celebrate! After all, an algorithm for the universe spells the end of science, the end of history, the end of surprises
- Look for another computational architecture

Cellular automata

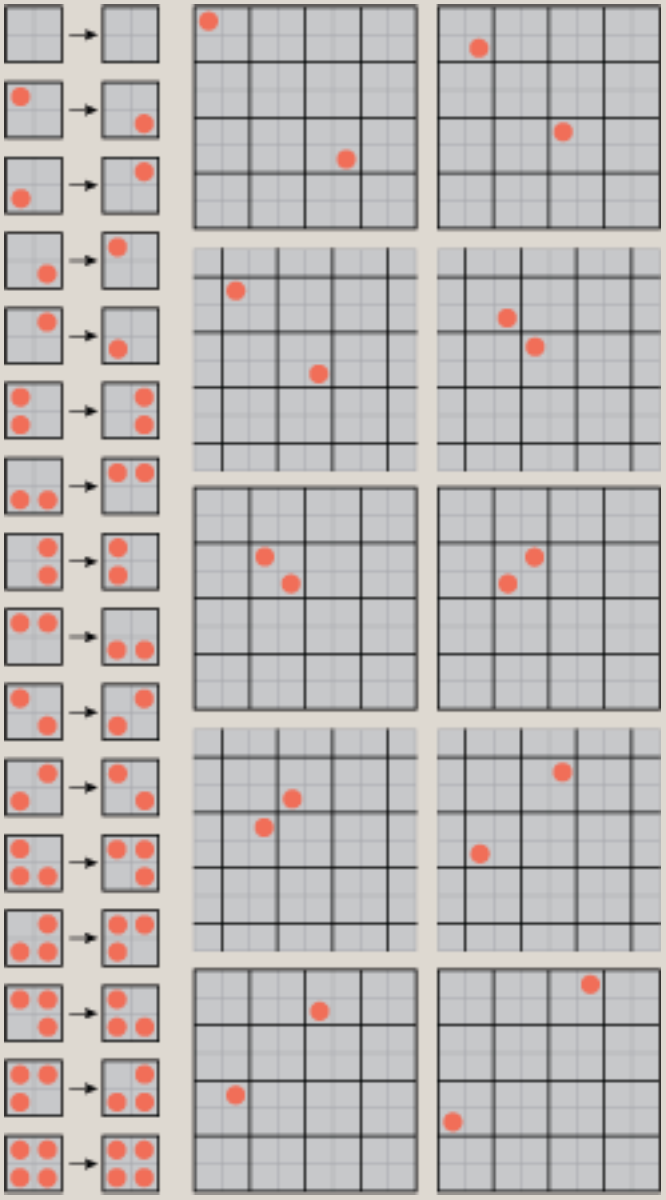


Cellular automata

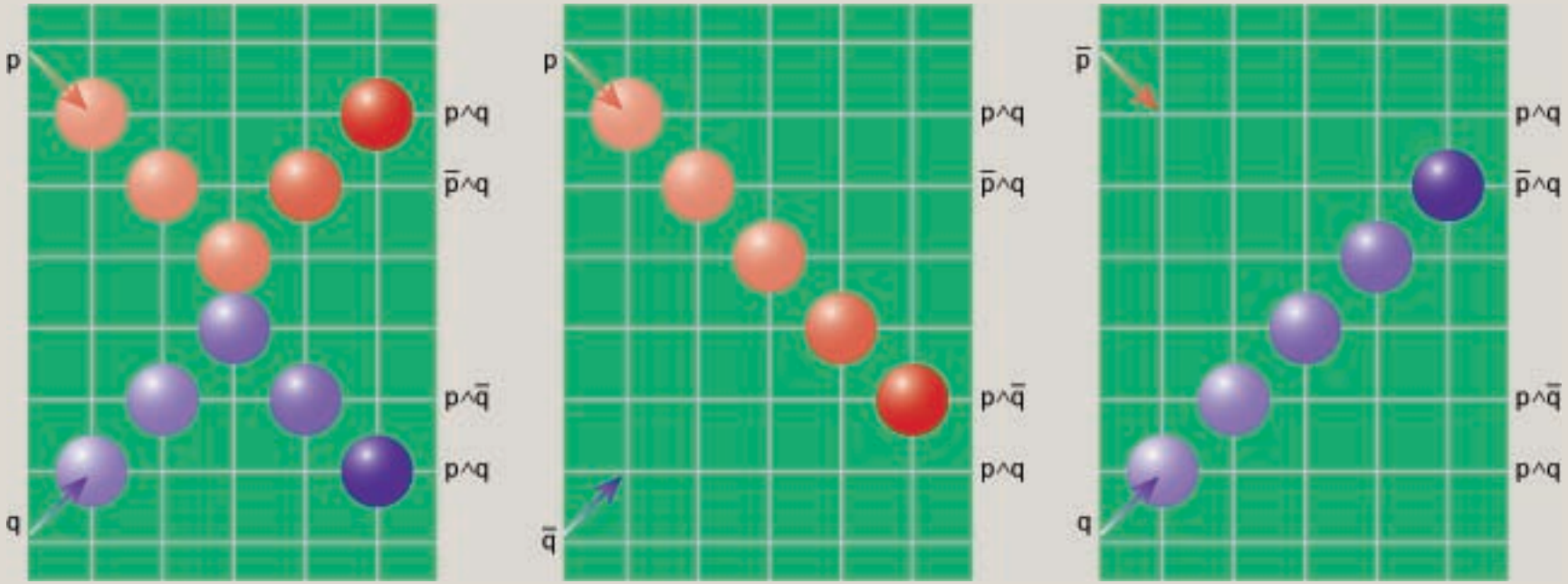
Instead of endowing every particle with a CPU, we let each point in space compute its next state

Pros Everything is local; uniform structure; computational resources do not vary with the content of the universe; discrete space and time

Cons Breaks rotational symmetry; details of the "instruction set" yet to be worked out; discrete space and time



The computational universe computes…



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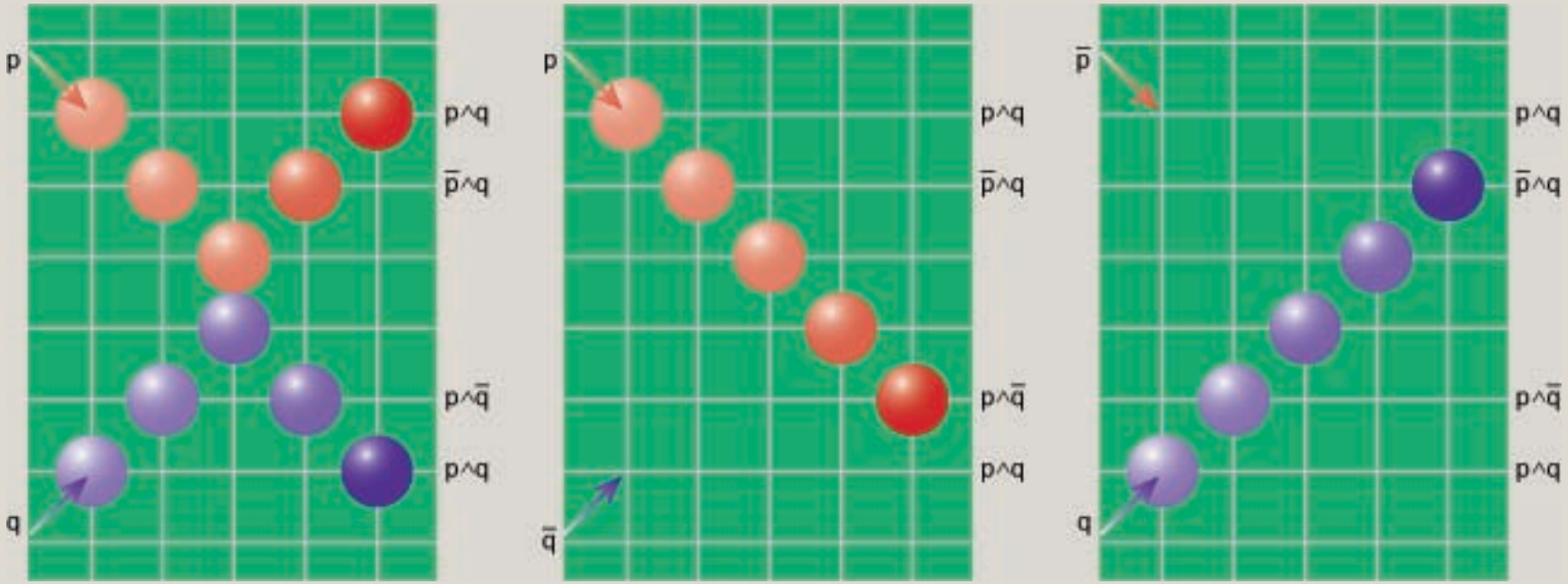

But would it have pleased Alfonso?

Thanks!

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