Gauss *vs.* The Virile Brute

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Pop Quiz!

$$1 + 2 + 3 + \ldots + 100 = \ ?$$

Pop Quiz!

$$1 + 2 + 3 + \ldots + 100 = 5{,}050$$

Gauss sums it up

An old story...

In the 1780s a provincial German schoolmaster gave his class the tedious assignment of summing the first 100 integers. The teacher's aim was to keep the kids quiet for half an hour, but one young pupil almost immediately produced an answer: $1 + 2 + 3 + \ldots + 100 = 5{,}050$. The smart aleck was Carl Friedrich Gauss, who would go on to join the short list of candidates for greatest mathematician ever.

… the rest of the story

Gauss was not a calculating prodigy who added up all those numbers in his head. He had a deeper insight: If you "fold" the series of numbers in the middle and add them in pairs—1+100, 2+99, 3+98, and so on—all the pairs sum to 101. There are 50 such pairs, and so the grand total is simply 50×101. The more general formula, for a list of consecutive numbers from 1 through n, is $n(n+1)/2$.

Questions

- Did the teacher also know the trick?
- If so, how could he assign the problem and not teach the method?
- If not, how did he solve the problem?
- How did all the other kids do it?

More questions

- Where did this story come from?
- How has it been passed down to us?
- Was it a factual incident, or more of a mythic tale, like Newton and the apple?
- How has the story evolved in its many retellings over the years?

Still more questions

- Why do we keep telling this story?
- What is its role in mathematical culture?
- What is its role in the classroom?
- Does it offer a useful lesson for all the kids in the class, not just young geniuses like Gauss?

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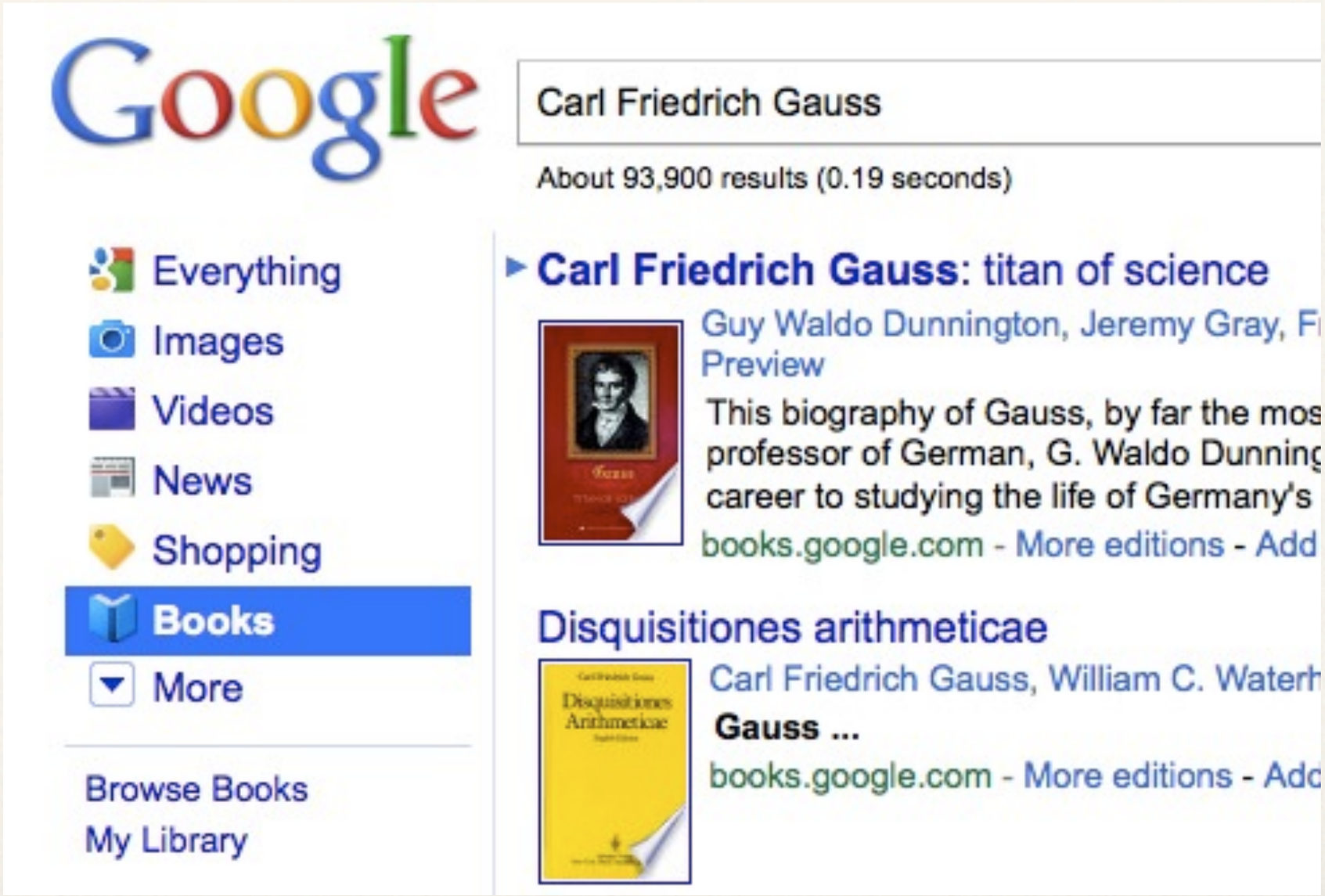
...and at the Library of Babble



The source

Wolfgang Sartorius,

Baron von Waltershausen,

(1809–1876)

Gauss Remembered

G A U S S

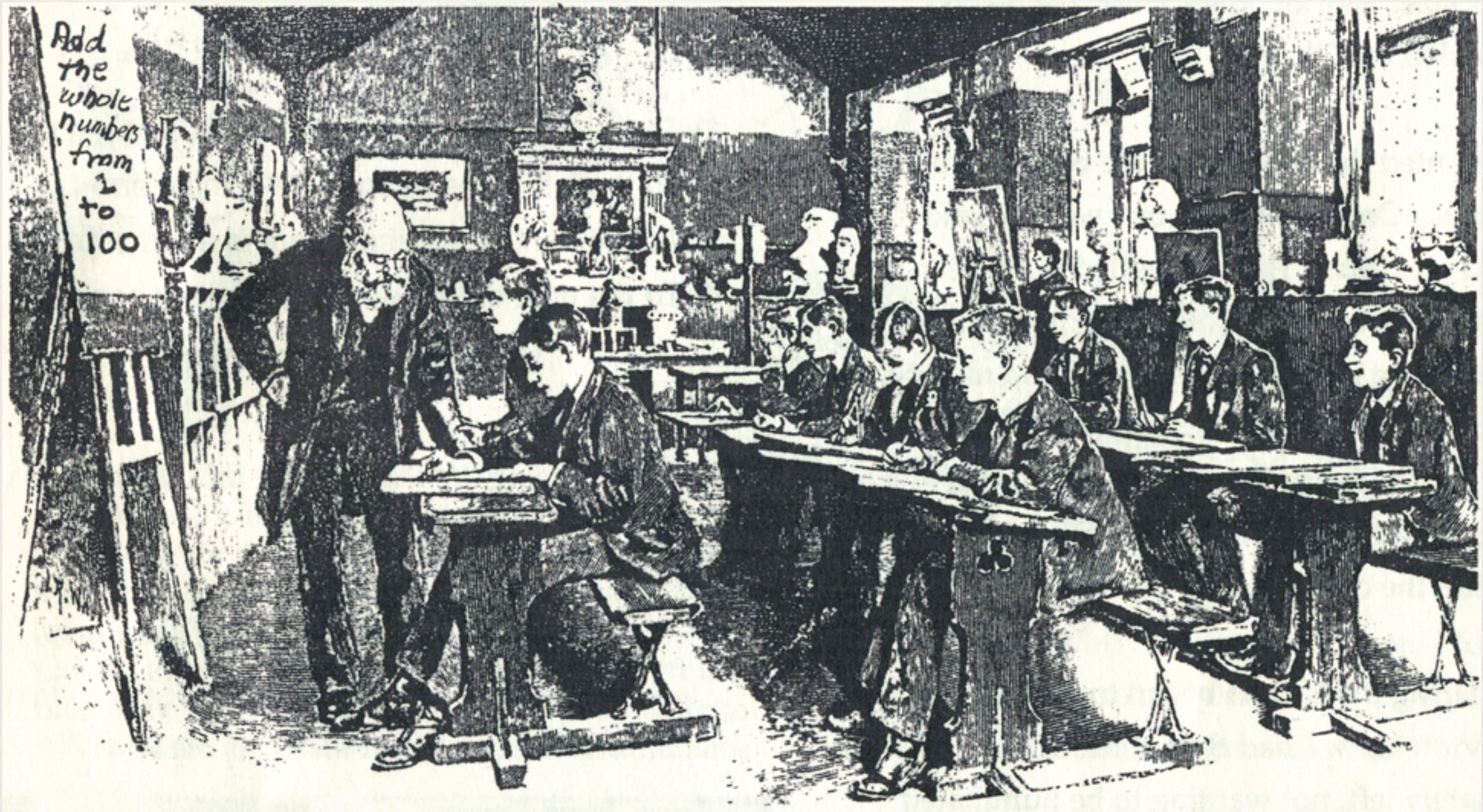
ZUM GEDÄCHTNISS.

VON

W. SARTORIUS v. WALTERSHAUSEN.

The Sartorius version (1)

In 1784 after his seventh birthday the little fellow entered the public school where elementary subjects were taught and which was then under a man named Büttner. It was a drab, low schoolroom with a worn, uneven floor....


Add the whole numbers from 1 to 100

The Sartorius version (2)

Here among some hundred pupils Büttner went back and forth, in his hand the switch which was then accepted by everyone as the final argument of the teacher. As occasion warranted he used it....

The Sartorius version (3)

> Here occurred an incident which [Gauss] often related in old age with amusement and relish. In this class the pupil who first finished his example in arithmetic was to place his slate in the middle of a large table. On top of this the second placed his slate and so on.

The Sartorius version (4)

The young Gauss had just entered the class when Büttner gave out for a problem the summing of an arithmetic series. The problem was barely stated before Gauss threw his slate on the table with the words (in the low Braunschweig dialect): “There it lies.”

The Sartorius version (5)

While the other pupils continued counting, multiplying and adding, Büttner, with conscious dignity, walked back and forth, occasionally throwing an ironical, pitying glance toward this the youngest of the pupils. The boy sat quietly with his task ended, as fully aware as he always was on finishing a task that the problem had been correctly solved and that there could be no other result.

The Sartorius version (6)

At the end of the hour the slates were turned bottom up.
That of the young Gauss with one solitary figure lay on top.
When Büttner read out the answer, to the surprise of all present that of young Gauss was found to be correct, whereas many of the others were wrong.

Recurrent themes

- The ritual of the slates.
- The brave declaration “There it lies!” (“Ligget se”).
- Büttner and his whip.

What's missing in Sartorius?

- The theme of "busy work" while the teacher takes a break.
- The specific series 1 + 2 + 3 + ... + 100 (or any other).
- How Gauss solved the problem.
- How Büttner solved the problem.
- The shortcut formula for summing an arithmetic series.

Is Sartorius the *only* source?

- I certainly can't prove it, and a new document could turn up.
- But if another source exists, it has had no influence on the known literature.
- All other 19th-century accounts echo aspects of Sartorius and appear to be derived from his memorial volume.

Whence 1+2+3+...+100?

- Earliest example known to me: 1 + 2 + 3 + ... + 40, published 1894.
- Franz Mathé, 1906, suggested 100 + 99 + 98 + ... + 1.
- Ludwig Bieberbach (1938) reversed the sequence: 1 + 2 + 3 + ... + 100.
- In 1937 Waldo Dunnington failed to mention any specific series, but in his 1955 biography he went along with 1 + 2 + 3 + ... + 100.
- But meanwhile in Eric Temple Bell was telling a different story....

The Eric Temple Bell version (1)

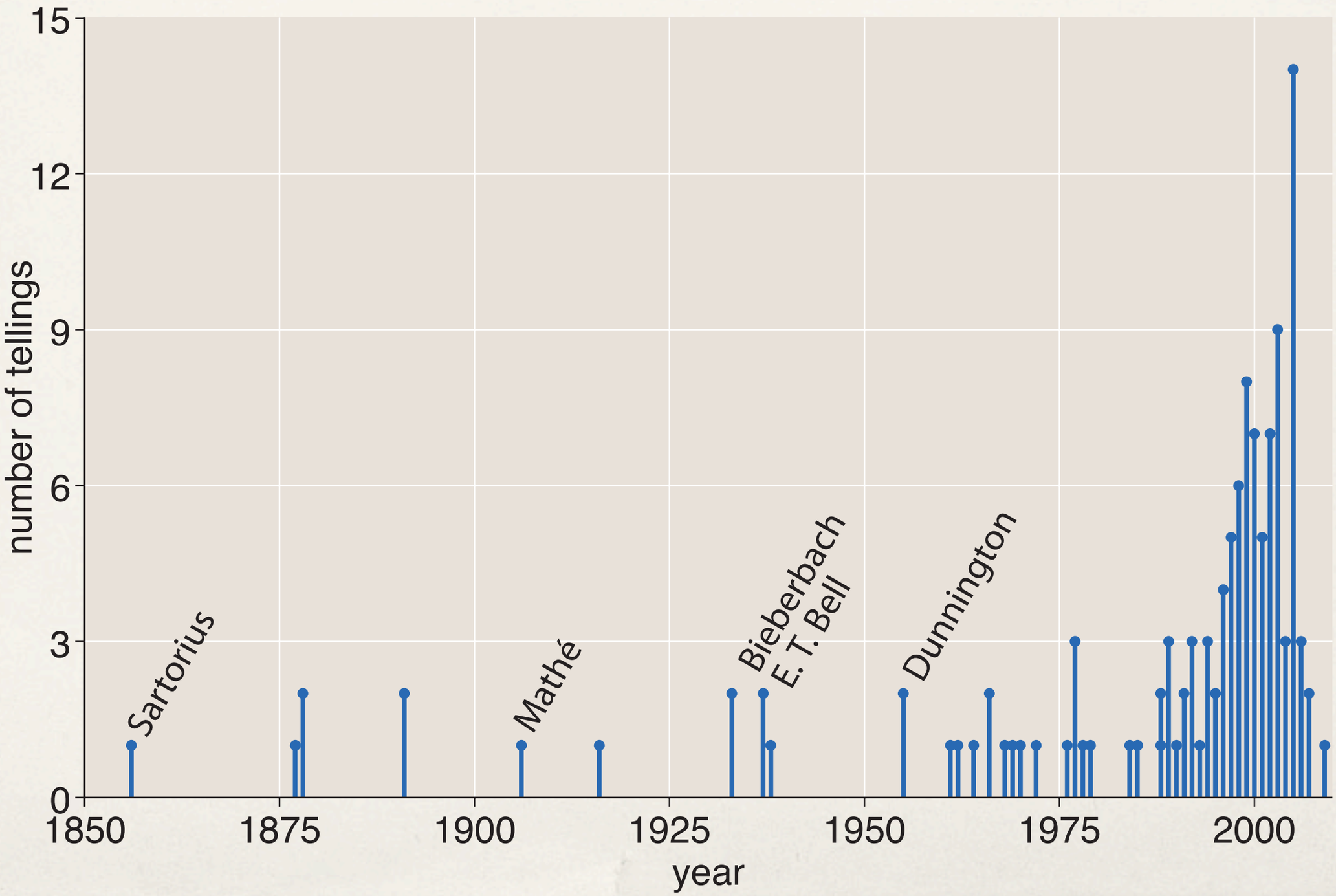
Shortly after his seventh birthday Gauss entered his first school, a squalid relic of the Middle Ages run by a virile brute, one Büttner, whose idea of teaching the hundred or so boys in his charge was to thrash them into such a state of terrified stupidity that they forgot their own names. . .

The Eric Temple Bell version (2)

None of the boys had ever heard of an arithmetic progression. It was easy then for the heroic Büttner to give out a long problem in addition whose answer he could find by a formula in a few seconds. The problem was of the following sort, 81297 + 81495 + 81693 + ... + 100899, where the step from one number to the next is the same all along (here 198), and a given number of terms (here 100) are to be added.

Proliferation, Mutation, Embroidery

Timeline


15
12
9
6
3
0
number of tellings
Sartorius
Mathé
Bieberbach
E. T. Bell
Dunnington
1850
1875
1900
1925
1950
1975
2000
year

Genres

- Biographies and histories; textbooks, class notes; children's books
- Collections of math lore and anecdotes
- Books on problem-solving
- Programming-language manuals; a physics text
- Student term papers
- A poem; a novel; a one-act play; a radio presentation
- A joke

Authors

- Keith Devlin
- Marcus De Sautoy
- Howard Eves
- Ron Graham, Don Knuth and Oren Patashnik
- Stephen Hawking
- T. W. Körner
- Stephen Krantz
- Eli Maor
- George Polya
- Ian Stewart
- Linus Torvalds

Any series will do...

$$1 + 2 + 3 + \cdots + 100 = 5050$$

$$1 + 2 + 3 + \cdots + 80 = 3240$$

$$1 + 2 + 3 + \cdots + 50 = 1275$$

$$1 + 2 + 3 + \cdots + 40 = 820$$

$$1 + 2 + 3 + \cdots + 20 = 210$$

$$1 + 2 + 3 + \cdots + 10 = 55$$

Any series will do...

$$1 + 2 + 3 + \cdots + 1000 = 500500$$

$$0 + 1 + 2 + \cdots + 100 = 5050$$

$$1 + 2 + 3 + \cdots + 99 = 4950$$

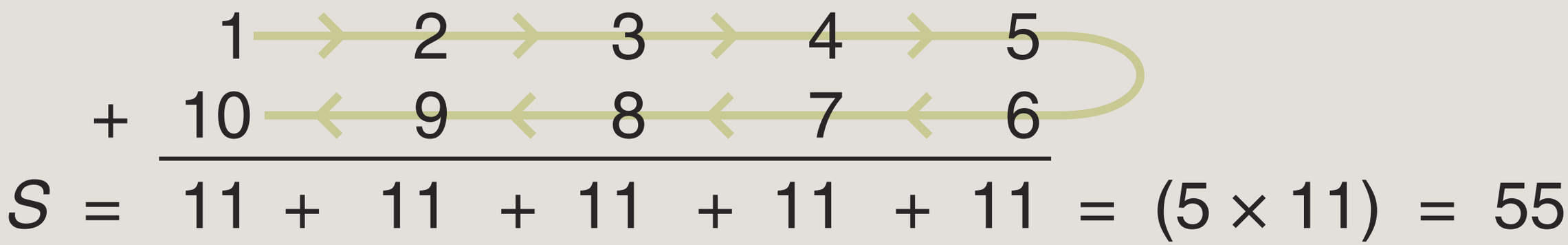
$$11 + 14 + 17 + \cdots + 26 = 111$$

$$3 + 7 + 11 + \cdots + 27 = 105$$

$$81297 + 81495 + \cdots + 100899 = 9109800$$

Any algorithm will do...

folding



Any algorithm will do...

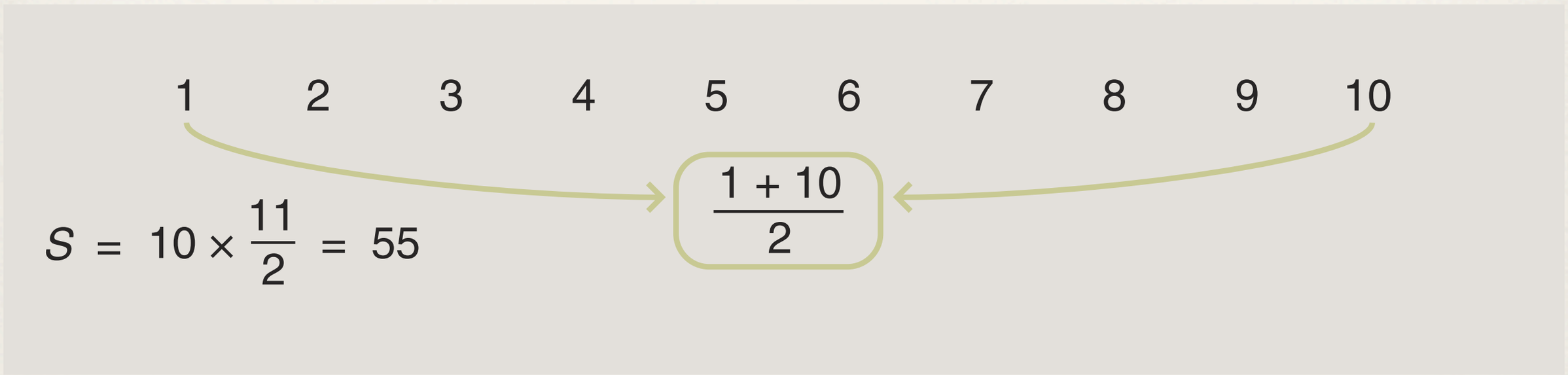
two rows

$$
\begin{array}{rccccccccccc}
 & 1 & \rightarrow & 2 & \rightarrow & 3 & \rightarrow & 4 & \rightarrow & 5 & \rightarrow & 6 & \rightarrow & 7 & \rightarrow & 8 & \rightarrow & 9 & \rightarrow & 10 \\
+ & 10 & \leftarrow & 9 & \leftarrow & 8 & \leftarrow & 7 & \leftarrow & 6 & \leftarrow & 5 & \leftarrow & 4 & \leftarrow & 3 & \leftarrow & 2 & \leftarrow & 1 \\
\hline
2S = & 11 & + & 11 & + & 11 & + & 11 & + & 11 & + & 11 & + & 11 & + & 11 & + & 11 & + & 11
\end{array}
= (10 \times 11) = 110
$$

$$S = 110 \div 2 = 55$$

Any algorithm will do...

average



Any formula will do...

$$\frac{n}{2}(n+1) \qquad \frac{n(n+1)}{2} \qquad n\frac{n+1}{2}$$

Whisper down the lane (1998)

At the age of ten, he was a show-off in arithmetic class at St. Catherine elementary school, "a squalid relic of the Middle Ages. . . run by a virile brute, one Büttner...." One day, as Büttner paced the room, rattan cane in hand, he asked the boys to find the sum of all the whole numbers from 1 to 100.

Whisper down the lane (2005)

It is well known among mathematicians... that Carl Friedrich Gauss, when he was ten years old—stunned his schoolteacher by performing the sum 1 + 2 + 3 + ... + 99 + 100—which the teacher had given the class in order to fill up the afternoon—in a minute or two... The story has, however, been transmogrified with time. It is thought that the actual sum that Gauss was asked to calculate was 81297 + 81495 + 81693 + ... + 100899.

Even Sartorius is transmogrified

Original: Der junge Gauss war kaum in die Rechenclasse eingetreten, als Büttner die Summation eine arithmetischen Reihe aufgab.

Translation: The young Gauss had just entered the class when Büttner gave out for a problem the adding of a series of numbers from 1 to 100.

(The translation is by Helen Worthington Gauss, great-granddaughter of C F. Gauss.)

What do we know?

- Probably not a fabrication: Something like this happened, but we'll never know the details.
- Classroom problems of this kind were common at the time.
- Büttner would surely have known the method.
- Whatever the historical source, the story has become a kind of proverb or fable. Factual accuracy is not the point.

But what about all the other kids?

“All this went through Gauss’s little head in a flash.”

- Some students listen to the story and think, “Cool. I’m gonna be like Gauss.”
- Some students hear the story and think, “I’d have been one of those poor schlubs who had to add up columns of numbers for an hour.”
- Math is not just for geniuses.
- Only one right answer, but many ways to find it.

How did the other 99 do it?

Just brute-force arithmetic?

- Sartorius: “Während die andern Schüler emsig weiter rechnen, multipliciren und addiren . . .”

- 1966 translation of Sartorius: “While the other pupils continued busily adding . . .”

1
2
3
4
5
6
7
8
9
10
11
12
.
.
.
99
100

$= 45$

.
.
.
10
11
12
13
14
15 = 10 x 1
16
17
18
19
20
.
.
.

.
.
.
20
21
22
23
24
25 = 10 x 2
26
27
28
29
30
.
.
.

.
.
.
30
31
32
33
34
35 = 10 x 3
36
37
38
39
40
.
.
.

$$(10 \times 45) + (10 \times (10 + 20 + 30 + 40 + 50 + 60 + 70 + 80 + 90)) + 100$$

$$= 5050$$

1	11	21	31	41
2	12	22	32	42
3	13	23	33	43
4	14	24	34	44
5	15	25	35	45
6	16	26	36	46
7	17	27	37	47
8	18	28	38	48
9	19	29	39	49
10	20	30	40	50
55	155	255	355	455

$$55 + 155 + 255 + 355 + 455 + 555 + 655 + 755 + 855 + 955$$

$$= 5050$$

Mathematics for pedestrians

- For most of us, mathematics is not just one brilliant flash of genius after another.
- Working at a problem, trying experiments, helps us find patterns.
- This is also a route to insight.

Büttner in the computer age

Not:

"Sum the integers from 1 to 100"

but:

"Write a program to sum the integers from 1 to *n*."

Gauss's way

```
function gsum(n)
  return n * (n + 1) / 2
```

The brute-force way

```
function bsum(n)
sum = 0; k = 1;
repeat
    sum += k;
    k += 1;
until k > n;
return sum
```

Which is better?

	`gsum`	`psum`
brevity	✓	
computer running time	✓	
programmer thinking time		✓
perspicacity		✓
likelihood of avoiding bugs		✓
numerical correctness		✓
ease of generalizing		✓

Sum of squares (Gauss)

```
function gsumSquares(n)
   return (n * (n + 1)
              * (2 * n + 1)) / 6
```

Sum of squares (brute force)

```
function bsum(n)
sum = 0; k = 1;
repeat
    sum += k * k;
    k += 1;
until k > n;
return sum
```

Web resources

http://bit-player.org/gauss-links

Thanks to my helpers

- Johannes Berg, University of Cologne
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- Barry Cipra
- Herb Acree

And thanks to you!

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