This Head Is False

GODEL, ESCHER, BACH

An Eternal Golden Braid.
By Douglas R. Hofstadter.
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By BRIAN HAYES

CERTAIN ideas in the sciences have been stuffed almost to bursting with metaphoric meaning. Everybody's favorite is the concept of entropy, a measure of disorder in thermodynamics. Entropy tends to increase, and so the word is called on to express a variety of sentiments about the common fate of dissipation and decay. The uncertainty principle of quantum mechanics has been extended, or distended, in a similar way: From the principle that any observer disturbs the thing he measures comes the notion that no bystander is entirely innocent.

The incompleteness theorem proved in 1931 by Kurt Gödel seems to be another candidate for metaphoric inflation. It is a great truth, and so it ought to have a large meaning; perhaps it should have the power to change lives. Unlike entropy and uncertainty, however, the incompleteness theorem is not the kind of idea that grabs you by the lapels and insists on being recognized.

The theorem is a variation on the only well-remembered line of the Cretan poet Epimenides, who said, "All Cretans are liars." Another version of the same antinomy is more succinct and more troublesome; it reads, "This sentence is false." The unsettling effect of these statements was for a long time attributed to the looseness and ambiguity of natural languages, where a phrase can refer simultaneously to more than one thing. It was assumed that in a formal language, one constructed on strict rules of logic, no such inconsistent statements could be formulated; they would be unutterable. Gödel showed otherwise.

Gödel's proof employs a formal language invented by Bertrand Russell and Alfred North Whitehead, who had set out to build a secure foundation for the arithmetic of whole numbers. The language has a vocabulary of symbols and a grammar of rules for combining the symbols to form "strings" which can be interpreted as statements about the properties of numbers. A few simple strings are accepted as axioms, or self-evident truths. Any string of symbols that can be derived from the axioms by applying the grammatical rules must also be true; it is therefore designated a theorem. The language is at once simple and powerful, and until 1931 it appeared to have the satisfying quality of completeness. Russell and Whitehead believed that any true property of the whole numbers could be demonstrated in their language, and that no false propositions could be proved.

The theorem by which Gödel upset that belief is a string of symbols in the Russell-Whitehead language that can be interpreted on two levels. In one sense it is a straightforward statement about the natural numbers that seems to be true; at the same time, it represents a statement of "metamathematics" with the evident meaning: "This string of symbols is not a theorem." The paradox of Epimenides is with us again, and this time there is no escaping through the loopholes of language. If the string can be derived from the axioms, then a falsehood has been proved and the Russell-Whitehead language is inconsistent; by implication, so is arithmetic. If the string cannot be derived from the axioms, then there is a true statement about the natural numbers that cannot be proved in the formal language. There is good reason for choosing the latter alternative and concluding that the Russell-Whitehead language is incomplete. In fact, the result is more general than that: Any system of formal logic powerful enough to describe the natural numbers is intrinsically incomplete.

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Elliott Banfield

It is easy enough to respond "So what?" No one thinks or speaks a formal language, and arithmetic seems to work quite well even if it is rotten at the core. Douglas R. Hofstadter, who is an assistant professor of computer science at Indiana University, addresses this issue at some length. At the heart of Gödel's theorem he finds the idea of self-reference, which can be viewed as a circular argument collapsed into itself. The same principle operates in other contexts, and in most of them it gives rise to no sensation of paradox.

In his title Professor Hofstadter yokes together Gödel, Johann Sebastian Bach and the Dutch artist Maurits Cornelis Escher, and a substantial part of his book is dedicated to showing that this is not such an unlikely team of oxen. Escher is the easier case: his drawings (like the paintings of René Magritte, which are also discussed) have an obvious connection with verbal and mathematical paradox. For example, the print "Waterfall" shows a mill race in which water seems to flow always downhill and yet moves from under the wheel to over it. The image is formally undecidable in the same way that Gödel's theorem is; the eye presents the mind with two competing interpretations, and neither one is fully satisfactory. Much other modern art plays a more obvious game of self-reference, asking whether the painting is a symbol or an object and frustrating any attempt to give a definitive answer.

The well-known combinatorial trickery of Bach's canons and fugues gives rise to another rich pattern of ambiguous perceptions. A theme enters, then appears again, inverted or reversed or in a different key or a different tempo; the transformed melody then blends with its original. Figure and ground may unexpectedly change roles. Even though each of the notes is heard distinctly — and in Bach the notes have a logic only slightly less formal than that of the Russell-Whitehead language — the ear cannot always resolve their relationship. Douglas Hofstadter would not argue that awareness of the underlying mathematics contributes much to appreciation of the music, but the music does illuminate the math. And, less seriously, there is at least one instance of explicit self-reference in Bach's work. In the last measures of the "Art of Fugue," written just before the composer died, he introduced a four-note melody that when transcribed in the German system of notation spells "B-A-C-H."

Escher and Bach are only the beginning of Professor Hofstadter's Shandean digressions. He traces connections that lead from Gödel's theorem to Zen, where contradiction is cherished; to the social insects, where it is not clear whether the ant or the entire colony should be regarded as the organism; to television cameras pointed at television screens; to "elemen-

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tary" particles of matter made up of still smaller elementary particles. He constructs a quite elaborate analogy between the incompleteness theorem and the transmission of genetic information encoded in the nucleotide sequences of DNA; here the self-reference of the theorem is comparable to the self-replication of the molecule. Most of all, he is at pains to present the implications of Gödel's proof for theories of the human mind (and of artificial intellects). In the mind the entire procedure of the Gödel proof seems to be repeated: A large but mechanistic, rule-following system, when it grows complex enough, develops the capacity for self-reference, which in this context is called consciousness.

Professor Hofstadter's presentation of these ideas is not rigorous, in the mathematical sense, but all the essential steps are there; the reader is not asked to accept results on authority or on faith. Nor is the narrative rigorous in the uphill-hiking sense, for the author is always ready to take the reader's hand and lead him through the thickets. Someone seeking no more than an introduction to Gödel's work would probably do better to look into a little book published 20 years ago by Ernest Nagel and James R. Newman, "Gödel's Proof," which is just as clear and thorough and is only one percent as long. But Douglas Hofstadter's book is a more ambitious project. It is also a more pretentious one. To accompany each expository chapter, the author has provided a whimsical dialogue cast in the form of a Bach composition. For example, one dialogue has the form of a canon cancrizans, which is the same when read forward or backward. Some may find these interludes amusing. For my part, I was strongly reminded that the challenge of writing such a piece is not in throwing the melodies together according to rule, but in making music of them. ■

Author's Query

I am editing the letters of Brendan Behan and would appreciate any useful material.

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