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about how this engaging, eminently readable tale was assembled.

The Shape of a Life does a splendid job of exploring the man, his mathematics, and the cultural milieu and intellectual and political setting in which he developed. The early chapters cover Yau's impoverished childhood in mainland China and later in Hong Kong, where his family fled in 1949, six months after his birth, amid the turmoil of China's civil war and the subsequent Communist takeover of the government. He vividly evokes the era and environment: crowded apartment buildings without electricity and running water, lack of money to buy food, and penalties for dress-code violations at school.

In 1969, at the height of campus protests against the Vietnam War, Yau arrived at the University of California, Berkeley, as a mathematics graduate student. There he found his general interest in mathematics turning specifically to geometry and nonlinear partial differential equations. "Somewhere in the back of my head," he writes, "I had vague notions about tying together geometry and topology, using partial differential equations as the connective thread." While attending some talks on general relativity in 1970, Yau first learned about the Ricci curvature tensor, a central term in Einstein's equations. Gregorio Ricci-Curbastro, the Italian mathematician for whom the tensor is named, had applied his tensor in 1904 to the study of the local distribution of curvatures in higher-dimensional spaces, and he gave it a geometrical interpretation. Yau's interest in Ricci curvature led directly to his interest in the Calabi conjecture, "both for its own sake and because of its relevance to general relativity," he recalls. He was convinced that the conjecture, whether true or not, was the key to understanding Ricci curvature. Soon he was captivated by the Calabi conjecture—he had "a gut-level, emotional response to [it]" that he didn't himself fully understand.

"The conjecture applies to spaces with a special kind of geometry," Yau explains. "Calabi had proposed a systematic strategy for constructing a vast number of manifolds endowed with special geometrical properties." But no one, including Yau, had ever seen a single example of such a manifold.

Yau took his preoccupation with the conjecture from Berkeley to the Institute for Advanced Study at Princeton, and from there to Stony Brook, to Stanford,

to New York University's Courant Institute, and finally to the University of California, Los Angeles, where in 1976 he arrived at a proof, having worked on the problem sporadically, sometimes to the point of physical exhaustion, for more than six years. The experience of mistakenly thinking some years earlier that he had disproved the conjecture weighed heavily on his mind. He recounts going over his new proof repeatedly at UCLA, "telling myself that if I got it wrong this time, I would give up mathematics altogether and try my hand at something different—maybe even duck farming." (An uncle had once offered to set him up in such a business.) For good measure, Yau sent a copy of his proof to Eugenio Calabi at the University of Pennsylvania, and a brief summary appeared the following year in the *Proceedings of the National Academy of Sciences of the U.S.A.* In 1982, Yau received the Fields Medal, the mathematical community's equivalent of the Nobel Prize, for this work.

Other technical topics that the book covers in detail are Yau's work in geometric analysis—a new branch of mathematics he helped get off the ground in connection with his Calabi proof—and his collaborations with physicists studying relativity. The book's 12 chapters also deal with his work on the positive mass conjecture; the story of mathematician Richard Hamilton, who discovered "Ricci flow" and proved a special case of the Poincaré conjecture; and the central role of Calabi-Yau manifolds in string theory, which aims to unite gravity with quantum theory.

For those with a taste for elegant and largely jargon-free explanations of mathematics, *The Shape of a Life* promises hours of rewarding reading. Readers will probably also enjoy Yau's revealing descriptions of academic politics, including candid accounts of bruising quarrels with colleagues, both in the United States and in China, where he has many ties and projects under way. By his own account, Yau has never felt quite at home in either country. His account of life in his natural home—mathematics—and his quest to uncover deep truths about nature proves to be a terrific read.

Judith Goodstein is a historian of mathematics and science, and an archivist. She is the author most recently of Einstein's Italian Mathematicians: Ricci, Levi-Civita, and the Birth of General Relativity.

Math with Attitude

MATH WITH BAD DRAWINGS: Illuminating the Ideas That Shape Our Reality. Ben Orlin. 367 pp. Black Dog and Leventhal, 2018. \$27.99.

Math books meant for a broad audience are often tinged with evangelical fervor. They yearn to reinspire all those millions who lost their faith in numbers somewhere between flash-card drills and the quadratic formula. Real mathematics isn't like that, the books assure us. Real mathematics is filled with exciting adventures: turning a sphere inside out without piercing the surface, tiling an infinite bathroom with a pattern that never repeats, drawing curves so squiggly they fill all of space, strolling around a Möbius band and returning as your own mirror image.

I have read and thoroughly enjoyed many books in this genre, and I've even written a couple of them myself. However, I've never really believed they are likely to convert anyone who's not already singing in the mathematical choir. The sad fact is, outside the circle of math enthusiasts, people aren't all that interested in sphere eversion and aperiodic tiling.

Ben Orlin's *Math with Bad Drawings* may have a better chance of reaching lost souls. Orlin has an advantage over ivory tower types like me. As a K–12 classroom teacher, he comes face-to-face with skeptical youth every day. When he asked a group of ninth graders why they study math, they settled on the answer, "to prove to colleges and employers that we are smart and hardworking." Orlin comments:

The students weren't wrong. Education has a competitive zero-sum aspect, in which math functions as a sorting mechanism. What they were missing—what I was failing to show them—was math's deeper function.

"Deeper function" is a revealing phrase. If I were writing that sentence, I might have said "math's deeper meaning" or perhaps "math's inner beauty." But Orlin is listening to his students, and they are telling him, "Keep your feet on the ground." In these pages there are no mind-boggling excursions into N -dimensional geometry or puzzles about self-referential sentences

that are true only if they're false. In this book, mathematics is a down-to-earth tool for describing and understanding the world, not an art form or a quest for esoteric truths. Orlin applies this tool to the activities of everyday life: rolling the dice, paying your taxes, rescuing the global economy from daredevil bankers, fixing the Electoral College, designing a Death Star for Galactic Emperor Palpatine. (I'll concede that Death Star engineering is not an everyday task for most of us, but even a math teacher deserves a little fun every now and then.)

The volume is organized in five parts. Part I is a brief introduction exploring what mathematics looks like to students and teachers as well as to mathematicians. Part II takes up geometry and design, praising the virtues of the triangle as a structural element and bravely taking on the contentious issue of A4 versus U.S. letter-size stationery. Part II is also where the Death Star turns up. Parts III and IV, comprising almost half the book, deal with probability and statistics: lotteries, baseball box scores, p -hacking in the sciences, and the curious practice of putting world literature through a statistical meat grinder. Part V turns to some economic and political themes.

As presented in *Math with Bad Drawings*, these topics require no mathematical knowledge or skills beyond the ken of a ninth grader—elementary arithmetic, some basic concepts in probability, enough geom-

etry to recognize a right triangle. It's ordinary schoolroom math—just the sort of thing that has bored and alienated generations of students. And yet Orlin spins it into a charming book you'll want to take to the beach, or at least keep handy by the commode.

What's his secret? Well, first of all, there are the bad drawings—although in truth they're not half bad. Not even quarter bad. Or maybe I'm just unusually susceptible to stick figures with oversize bubbleheads, whose eyes communicate a surprising gamut of human emotions. The expressive eyes sometimes migrate to other objects—polygons, coffee cups, gemstones, maps of Minnesota—where they are just as endearing. I want them in all my math books from now on, please.

The prose is also chipper and cheerful. I'll content myself with a single example, which happens to address one of the main messages of both the text and the bad drawings:

Fables and math have a lot in common. Both come from dusty, moth-eaten books. Both are inflicted upon children. And both seek to explain the world through radical acts of simplification.

If you want to reckon with the full idiosyncrasy and complexity of life, look elsewhere. Ask a biologist, or a painter of photorealistic landscapes, or someone who files their own taxes. Fable tellers

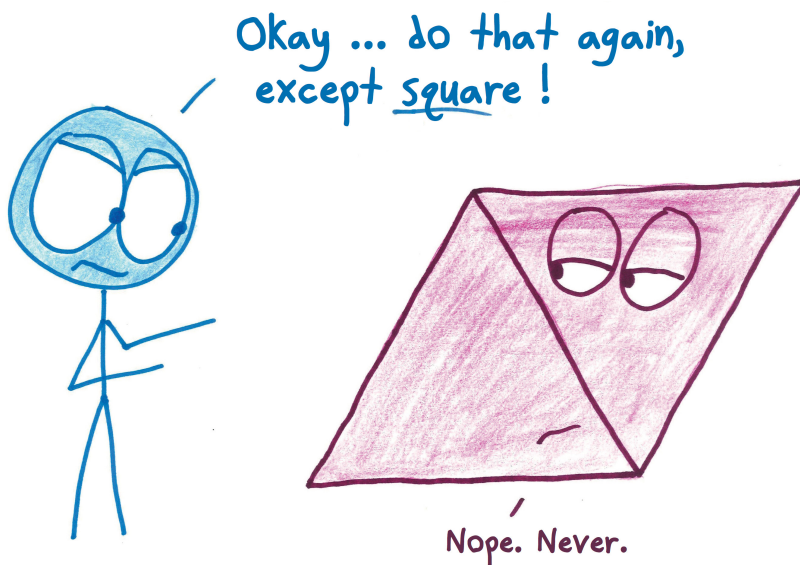
and math makers are more like cartoonists. By exaggerating a few features and neglecting all the rest, they help explain why our world is the way it is.

Orlin has a third secret ingredient, but it's invisible; it's something that's been deliberately left out of the recipe. "Do the math" and "show your work" are phrases that never turn up in these pages. There are no homework problems, no exercises for the reader, not even worked examples. The focus is on concepts, not algorithms or formulas or equations. Orlin occasionally gives the result of a numerical calculation, but he doesn't dwell on where the answer came from or explain how one might tackle similar problems. This mode of discourse would not be at all unusual in a work of history or literary criticism, but it's a radical departure in mathematics, where learning by doing is a way of life, and problem-solving is both a pastime and a rite of passage.

I was a few chapters into the book before I became fully conscious of this curious absence. My first reaction was: "No! Wait! You can't do that. You can't write a math book with no math in it." But why not? Authors in other disciplines are under no such compulsion. A book on music doesn't have to teach you how to play the guitar or compose a string quartet. Likewise not all books about food are full of recipes. Why should reading mathematics always be a roll-up-your-sleeves participatory process? As Orlin demonstrates, it's entirely possible to say interesting things about mathematics without showing people how to do mathematics. And this more discursive approach may help bring the gospel to an audience that would be turned away by scary notation.

If I haven't quite convinced you of the wisdom of mathless math writing, that's because I haven't quite convinced myself either. After all, mathematical notation has a purpose: It clearly expresses ideas that would be hard to communicate without it. Consider a passage in the introduction to the section on probability. After noting that the outcome of a single coin toss is 50/50, Orlin writes:

But if you could flip a trillion coins, you'd find yourself approaching a different world altogether: a well-groomed land of



"Say you're creating a square, and you want its diagonal to be the same length as its sides." Ben Orlin proposes this geometric impossibility, only to have it vetoed by the personification of a rhombus in one of his "bad drawings." From *Math with Bad Drawings*.

long-term averages. Here, half of all coins land on heads . . . and one-in-a-million events happen a millionth of the time, give or take.

These statements convey a deep truth: that random events in large enough numbers converge toward their average or expected behavior. Nevertheless, I worry that some readers will come away with a mistaken intuition about that experiment with a trillion coins. In particular, what is the probability of seeing exactly equal numbers of heads and tails? The phrase “half of all coins land on heads” might be taken to imply that the probability of this outcome grows larger as the number of coins increases, and that heads=tails would become a certainty with infinitely many coins. In fact, the probability of observing equally many heads and tails in a trillion coin flips is less than one in a million, and as the number of flips goes to infinity, the probability of an equal division wilts away to zero.

My point is not that Orlin’s statement about long-term averages is incorrect; at worst it’s slightly imprecise or incomplete. My point is that full understanding of a mathematical fact is hard to attain without doing some mathematics. Stands to reason, no? But then I see one of Orlin’s sleepy-eyed stick figures demanding, “Okay. Show me the math.” I’ll give it a try.

Allow me to start with an easier problem: the probability of getting equal numbers of heads and tails when flipping 100 coins rather than a trillion. The number of possible head-tail sequences in 100 coin flips is 2^{100} . How many of those sequences have exactly 50 heads and 50 tails? The answer is $100! / (50! \times 50!)$, where the exclamation point denotes the factorial function: $100! = 100 \times 99 \times 98 \times \dots \times 3 \times 2 \times 1$. Stacking up all those multiplications produces some very large numbers, but with computer assistance it’s not hard to calculate them. The probability we’re seeking is the number of 50-head sequences divided by the total number of sequences; it’s about 0.08.

To be thorough I would have to explain where those formulas came from and why you should believe they give the right answer, but I’m not going to bother, because the formulas are useless for the full-scale computation anyway. The number of possible outcomes when you flip a trillion coins is 2 raised to the trillionth power, which

is a number with too many bits to fit in my computer’s memory. To complete the computation I must resort to shortcuts or stratagems, such as working with logarithms of factorials. With some algebraic hocus-pocus, the formula for the probability of equal heads and tails can be reduced to a remarkably simple approximation: $1/\sqrt{\pi n}/2$, where n is the number of coins being flipped. For $n=1$ trillion, this works out to 8×10^{-7} . The challenge, of course, is explaining the hocus-pocus. Perhaps I could do so in terms the stick figure would understand, but it would take at least a few paragraphs, and I’m sure those droopy eyes would close before I could finish.

Euclid supposedly declared, “There is no royal road to geometry.” He was scolding an overprivileged pupil who was tired of ruts and potholes and wanted a well-paved route to the summit of knowledge. Orlin hasn’t built the royal road, but he’s offering aerial tours of the mountainside that are well worth taking. The details may be hard to discern from this altitude, but the scenery is great! I look forward to the sequel, although I am disappointed to learn it will not be titled *More Math with Worse Drawings*.

Brian Hayes is a former editor and columnist for American Scientist. His most recent book is Foolproof, and Other Mathematical Meditations.

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Finding Hope in a Dying Forest

IN SEARCH OF THE CANARY TREE: The Story of a Scientist, a Cypress, and a Changing World. Lauren E. Oakes. 272 pp. Basic Books, 2018. \$27.

Ecologist Lauren Oakes’s memoir *In Search of the Canary Tree* describes the six years she spent studying a tree that lives on the coasts of the Pacific Northwest: *Callitropsis nootkatensis*, whose common names include yellow cypress and yellow cedar. Because the yellow cedar population has declined precipitously with the progression of climate change, Oakes likens the tree to a canary in a coal mine, a comparison alluded to in the book’s title. Her research investigated what kinds of trees are grow-

ing in to replace the dead and dying cypress. While she was immersed in this work, her father passed away unexpectedly, so she is grappling with loss on multiple fronts. Refusing to back away from topics shrouded in grief and fear, she describes herself as “looking for hope in a graveyard.”

Her initial study question was simple enough: “I wanted to know what species could still thrive amidst loss and change; what life could tolerate the conditions we’re creating, and how and why.” The fieldwork she describes is grueling. She and her two field assistants, nicknamed P-Fisch and Maddog, have to wait until the weather is right and then fly or boat in to sites in the Alexander Archipelago off the coast of Alaska, where the trees they are studying grow. They camp for weeks at a time, traveling by kayak to their field sites. Conditions are generally wet and chilly; there is no cell phone service, and satellite phone coverage is spotty.

Oakes’s fieldwork is punctuated by the self-doubt every new researcher feels as she encounters new problems. She describes candidly the struggles of a doctoral student in ecology: the hardship of the field, the painstaking and time-consuming work, the uncertainties, the unanticipated demands, the difficult decisions, the fatigue. And she also describes the exuberance she feels when these challenges are overcome.

But Oakes stands apart in that she wants to do more than simply study the consequences of environmental changes. “Most scientists today show graphs and numbers, complicated models, and statistics that basically say, ‘We are too late,’” she notes. She wanted in her dissertation research to blend ecology and social science, using the latter to search “for a way out of my own sense of fear and helplessness.” By interviewing the Alaskans who were losing this tree species—which they used, valued, and loved—she found out how they were responding to these changes.

But after completing and defending her dissertation, Oakes didn’t feel closure. “Instead,” she writes, “something felt unresolved, and it was far more personal than scientific.” She had failed to include the humanity of those she interviewed. “In distilling 1,500 pages of interview transcripts into a single, elegant table,” she says, “I’d left out the way a logger runs his calloused hand across fine-grained wood in admiration, or the silence that fills the room